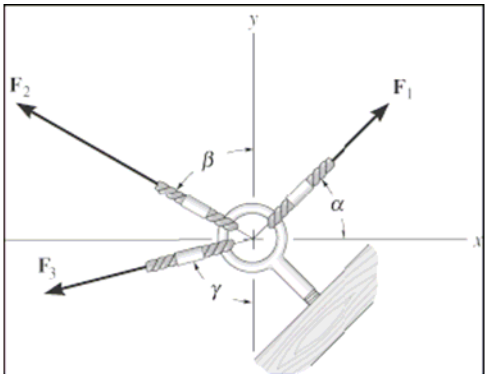
Problem 1
Determine the magnitude of the resultant force $\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3}$ and its direction, measured counterclockwise from the positive *x-axis*.

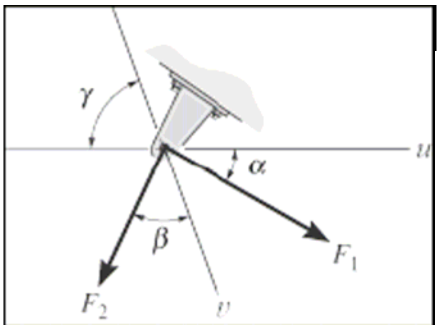


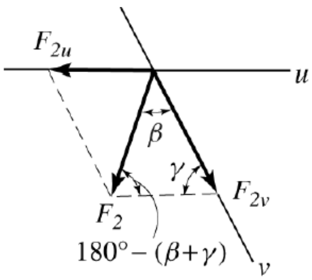
$\mathbf{F_{Rx}} = \mathbf{F_{1x}} + \mathbf{F_{2x}} + \mathbf{F_{3x}} = \mathbf{-\ 0.703\ kN}$, $\mathbf{F_{Ry}} = \mathbf{F_{1y}} + \mathbf{F_{2y}} + \mathbf{F_{3y}} = \mathbf{0.708\ kN}$

$\mathbf{F_R} = \mathbf{0.998\ kN}$ $\theta = 134.8^{o}$

Given:
$F_1 = 600$ N $F_2 = 800$ N $F_3 = 450$ N
$\alpha = 45$ deg m $\beta = 60$ deg $\gamma = 75$ deg

Problem 2
Resolve the force $\mathbf{F_2}$ into components acting along the *u* and *v* axes and determine the magnitudes of the components.





$$F_{2u} = F_2\left(\frac{\sin(\beta)}{\sin(\gamma)}\right)$$

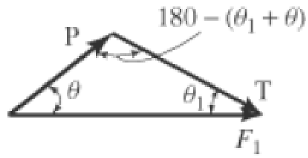
$$F_{2u} = 376.2\ \text{N}$$

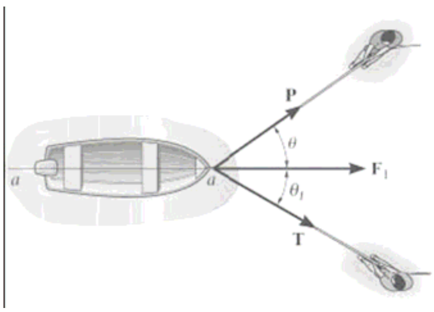
$$F_{2v} = F_2\left[\frac{\sin[180\ \text{deg} - (\beta + \gamma)]}{\sin(\gamma)}\right]$$

$$F_{2v} = 482.2\ \text{N}$$

Given:
$F_1 = 300$ N $F_2 = 500$ N $\alpha = 30$ deg $\beta = 45$ deg $\gamma = 70$ deg

Problem 3
The boat is to be pulled onto the shore using two ropes. Determine the magnitudes of forces T and **P** acting in each rope in order to develop a resultant force $\mathbf{F_1}$, directed along the keel axis *aa* as shown.





$$\frac{F_1}{\sin[180\ \text{deg} - (\theta + \theta_1)]} = \frac{T}{\sin(\theta)}$$

$$T = 54.7\ \text{lb}$$

$$T = F_1\frac{\sin(\theta)}{\sin(180\ \text{deg} - \theta - \theta_1)}$$

$$\frac{F_1}{\sin[180\ \text{deg} - (\theta + \theta_1)]} = \frac{P}{\sin(\theta_1)}$$

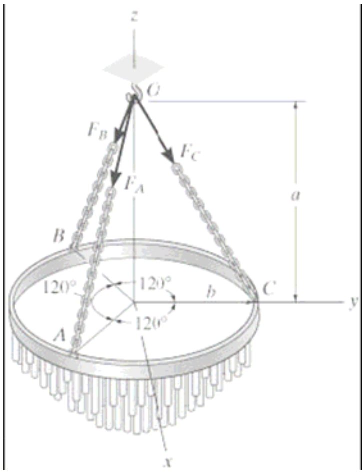
$$P = \sin(\theta_1)\frac{F_1}{\sin[180\ \text{deg} - (\theta + \theta_1)]}$$

$$P = 42.6\ \text{lb}$$

Given:
$\theta = 40$ deg $\theta_1 = 30$ deg $F_1 = 80$ lb

Problem 4

The chandelier is supported by three chains, which are concurrent at point *O*. If the resultant force at *O* has magnitude F_R and is directed along the negative *z-axis*, determine the force in each chain assuming $F_A = F_B = F_C = F$.



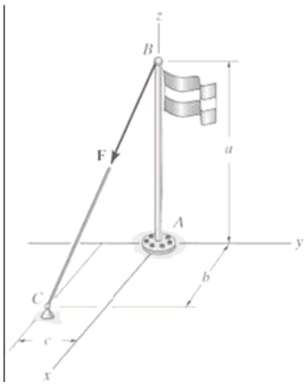
$$F = \frac{\sqrt{a^2 + b^2}}{3a} F_R$$

$$F = 52.1 \text{ lb}$$

$a = 6$ ft $\quad b = 4$ ft $\quad F_R = 130$ lb

Problem 5

Cable *BC* exerts force **F** on the top of the flagpole. Determine the projection of this force along the *z-axis* of the pole.



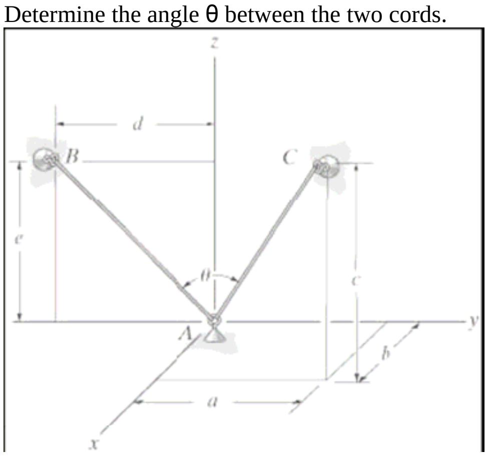
$$\mathbf{r_{BC}} = \begin{pmatrix} b \\ -c \\ -a \end{pmatrix} \qquad \mathbf{F} = F\frac{\mathbf{r_{BC}}}{|\mathbf{r_{BC}}|} \qquad F_z = -\mathbf{F} \bullet \mathbf{k} \qquad F_z = 24 \text{ N}$$

Given:

$F = 28$ N $\quad a = 12$ m $\quad b = 6$ m $\quad c = 4$ m

Problem 6

Determine the angle θ between the two cords.



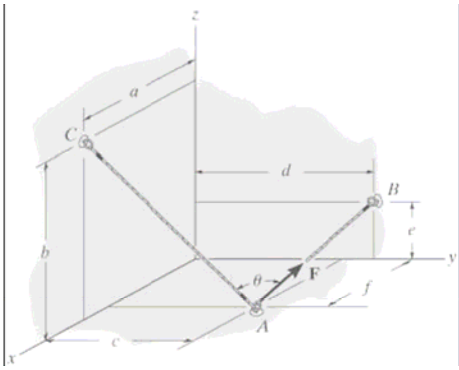
$$\mathbf{r_{AC}} = \begin{pmatrix} b \\ a \\ c \end{pmatrix} \text{ ft} \qquad \mathbf{r_{AB}} = \begin{pmatrix} 0 \\ -d \\ e \end{pmatrix} \text{ ft} \qquad \theta = \operatorname{acos}\left(\frac{\mathbf{r_{AC}} \cdot \mathbf{r_{AB}}}{|\mathbf{r_{AC}}|\,|\mathbf{r_{AB}}|}\right) \qquad \theta = 64.6 \text{ deg}$$

Given:

$a = 3$ m $\quad b = 2$ m $\quad c = 6$ m $\quad d = 3$ m $\quad e = 4$ m

Problem 7

Determine the projected component of the force **F** acting in the direction of cable *AC*. Express the result as a Cartesian vector.



$$\mathbf{r_{AC}} = \begin{pmatrix} a-f \\ -c \\ b \end{pmatrix} \text{m} \qquad \mathbf{u_{AC}} = \frac{\mathbf{r_{AC}}}{|\mathbf{r_{AC}}|} \qquad \mathbf{u_{AC}} = \begin{pmatrix} 0.2 \\ -0.6 \\ 0.8 \end{pmatrix}$$

$$\mathbf{r_{AB}} = \begin{pmatrix} -f \\ d-c \\ e \end{pmatrix} \qquad \mathbf{F_{AB}} = F\frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \qquad \mathbf{F_{AB}} = \begin{pmatrix} -9.6 \\ 3.2 \\ 6.4 \end{pmatrix} \text{lb}$$

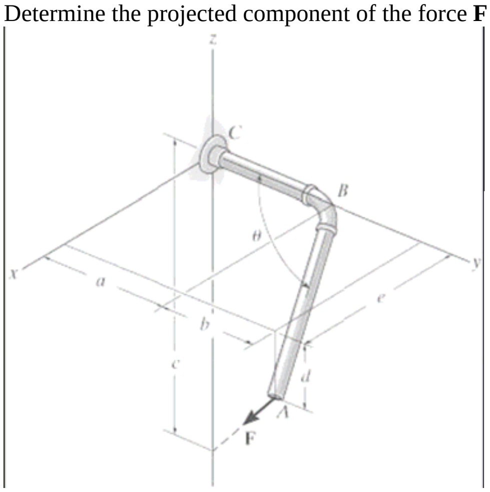
$$\mathbf{F_{AC}} = (\mathbf{F_{AB}} \cdot \mathbf{u_{AC}})\,\mathbf{u_{AC}} \qquad \mathbf{F_{AC}} = \begin{pmatrix} 0.229 \\ -0.916 \\ 1.145 \end{pmatrix} \text{lb}$$

Given:

F = 12 lb a = 8 ft b = 10 ft c = 8 ft d = 10 ft e = 4 ft f = 6 ft

Problem 8

Determine the projected component of the force **F** acting along the axis *AB* of the pipe.



$$\mathbf{r_A} = \begin{pmatrix} -e \\ -a-b \\ d-c \end{pmatrix} \quad \mathbf{r_A} = \begin{pmatrix} -6 \\ -7 \\ -10 \end{pmatrix} \text{m} \quad \mathbf{F} = F\frac{\mathbf{r_A}}{|\mathbf{r_A}|} \quad \mathbf{F} = \begin{pmatrix} -35.3 \\ -41.2 \\ -58.8 \end{pmatrix} \text{N}$$

$$\mathbf{r_{AB}} = \begin{pmatrix} -e \\ -b \\ d \end{pmatrix} \quad \mathbf{r_{AB}} = \begin{pmatrix} -6 \\ -3 \\ 2 \end{pmatrix} \text{m} \quad \mathbf{u_{AB}} = \frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \quad \mathbf{u_{AB}} = \begin{pmatrix} -0.9 \\ -0.4 \\ 0.3 \end{pmatrix}$$

Now find the projection using the Dot product.

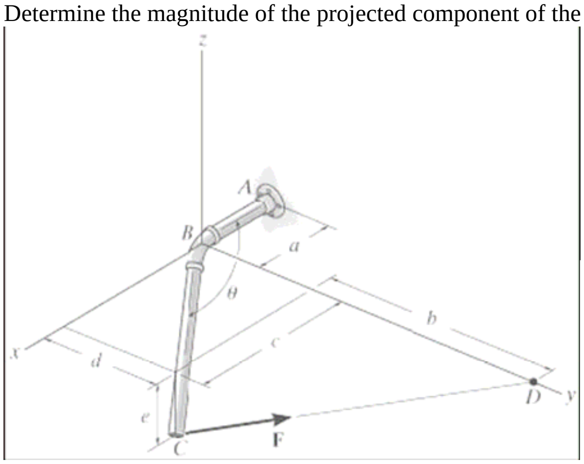
$$F_{AB} = \mathbf{F} \cdot \mathbf{u_{AB}} \qquad F_{AB} = 31.1 \text{ N}$$

Given:

F = 80 N a = 4 m b = 3 m c = 12 m d = 2 m e = 6 m

Problem 9

Determine the magnitude of the projected component of the force **F** acting along the axis *BC* of the pipe.



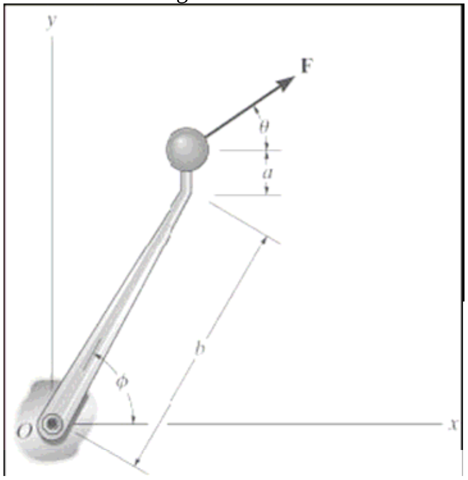
$$\mathbf{r_{CD}} = \begin{pmatrix} -c \\ b \\ e \end{pmatrix} \qquad \mathbf{u_{CD}} = \frac{\mathbf{r_{CD}}}{|\mathbf{r_{CD}}|} \qquad \mathbf{r_{CB}} = \begin{pmatrix} -c \\ -d \\ e \end{pmatrix} \qquad \mathbf{u_{CB}} = \frac{\mathbf{r_{CB}}}{|\mathbf{r_{CB}}|}$$

$$F_{BC} = (F\mathbf{u_{CD}}) \cdot \mathbf{u_{CB}} \qquad F_{BC} = 10.5 \text{ lb}$$

Given:

F = 100 lb a = 3 ft b = 8 ft c = 6 ft d = 4 ft e = 2 ft

Problem 10
Determine the magnitude of the force **F** that should be applied at the end of the lever such that this force creates a clockwise moment *M about* point *O*.

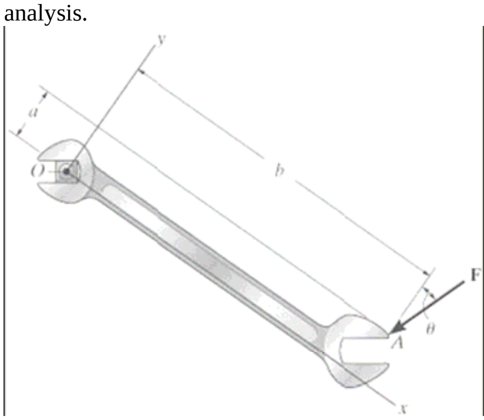


$$+\circlearrowleft\ M = F\cos(\theta)(a + b\sin(\phi)) - F\sin(\theta)(b\cos(\phi))$$

$$F = \frac{M}{\cos(\theta)(a + b\sin(\phi)) - \sin(\theta)(b\cos(\phi))} \qquad F = 77.6\ \text{N}$$

Given:
M = 15 Nm φ = 60 deg θ = 30 deg a = 50 mm b = 300 mm

Problem 11
A force **F** is applied to the wrench. Determine the moment of this force about point *O*. Solve the problem using both a scalar analysis and a vector analysis.



Scalar Solution

$$+\circlearrowleft\ \mathbf{M_O} = -F\cos(\theta)\,b + F\sin(\theta)\,a$$

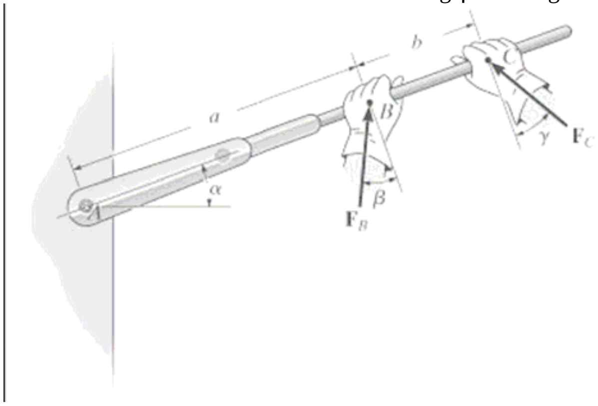
$$\mathbf{M_O} = -7.11\ \text{N·m} \qquad |\mathbf{M_O}| = 7.11\ \text{N·m}$$

Vector Solution:

$$\mathbf{M_O} = \begin{pmatrix} b \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} -F\sin(\theta) \\ -F\cos(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{M_O} = \begin{pmatrix} 0 \\ 0 \\ -7.11 \end{pmatrix}\text{N·m} \qquad |\mathbf{M_O}| = 7.107\ \text{N·m}$$

Given:
F = 40 N θ = 20 deg a = 30 mm b = 200 mm

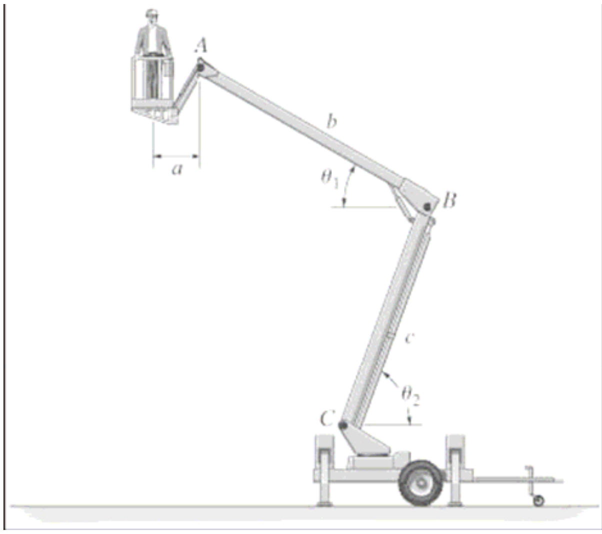
Problem 12
Determine the moment of each force about the bolt located at *A*.
Given: F_B = 40 lb a = 2.5 ft α = 20 deg γ = 30 deg F_C = 50 lb b = 0.75 ft β = 25 deg



$$\circlearrowright+\ M_B = F_B\cos(\beta)a \qquad \circlearrowright+\ M_C = F_C\cos(\gamma)(a + b)$$

$$M_B = 90.6\ \text{lb·ft} \qquad M_C = 141\ \text{lb·ft}$$

Problem 13
The Snorkel Co. produces the articulating boom platform that can support weight W. If the boom is in the position shown, determine the moment of this force about points A, B, and C.



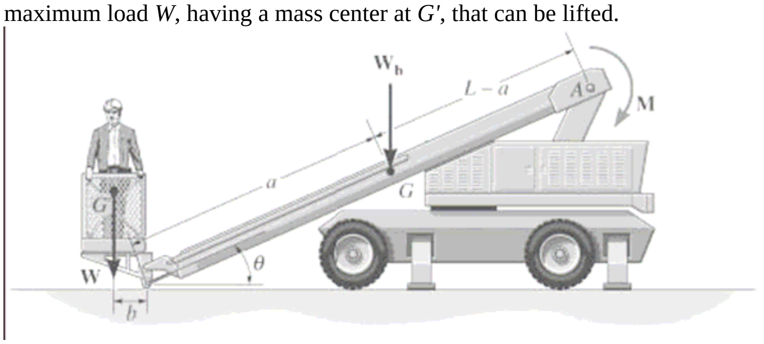
$$M_A = W a \qquad M_A = 1.65\ \text{kip}\cdot\text{ft}$$

$$M_B = W\left(a + b\cos(\theta_1)\right) \qquad M_B = 9.27\ \text{kip}\cdot\text{ft}$$

$$M_C = W\left(a + b\cos(\theta_1) - c\cos(\theta_2)\right) \qquad M_C = 6.45\ \text{kip}\cdot\text{ft}$$

Given:
$a = 3$ ft $b = 16$ ft $c = 15$ ft $\theta_1 = 30$ deg $\theta_2 = 70$ deg $W = 550$ lb

Problem 14
The boom has length L; weight W_b, and mass center at G. If the maximum moment that can be developed by the motor at A is M, determine the maximum load W, having a mass center at G', that can be lifted.

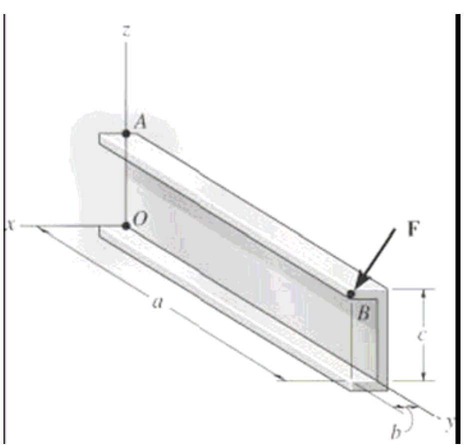


$$M = W_b\,(L - a)\cos(\theta) + W\,(L\cos(\theta) + b)$$

$$W = \frac{M - W_b\,(L - a)\cos(\theta)}{L\cos(\theta) + b} \qquad W = 319\ \text{lb}$$

Given:
$L = 30$ ft $W_b = 800$ lb $a = 14$ ft $b = 2$ ft $\theta = 30$ deg $M = 20 \times 10^3$ lb ft

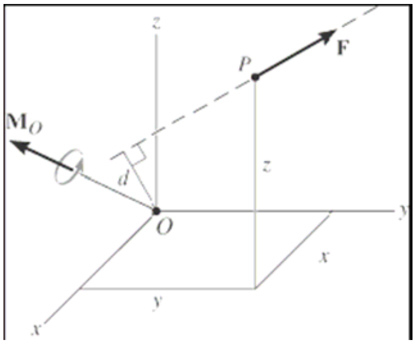
Problem 15
The force **F** acts at the end of the beam. Determine the moment of the force about point A. a) By vector method, b) By scalar method.



$$\mathbf{r_{AB}} = \begin{pmatrix} b \\ a \\ 0 \end{pmatrix} \qquad \mathbf{M_A} = \mathbf{r_{AB}} \times \mathbf{F} \qquad \mathbf{M_A} = \begin{pmatrix} -720 \\ 120 \\ -660 \end{pmatrix} \text{N}\cdot\text{m}$$

F:[600,300,−600] N
a= 1.2 m b= 0.2 m c= 0.4 m

Problem 16
The force **F** creates a moment about point O of $\mathbf{M}_O$. If the force passes through a point having the given x coordinate, determine the y and z coordinates of the point. Also, realizing that $M_O = Fd$, determine the perpendicular distance $\boldsymbol{d}$ from point $\boldsymbol{O}$ to the line of action of **F**.

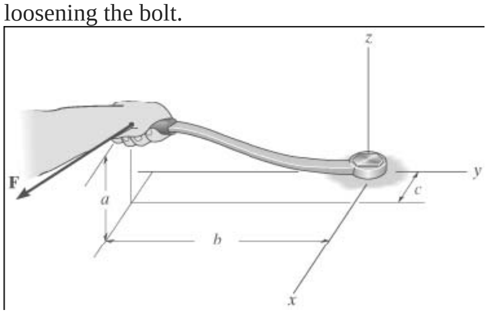


$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \times \mathbf{F} = \mathbf{M_O} \qquad \begin{pmatrix} y \\ z \end{pmatrix} = \text{Find}(y, z) \qquad \begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \text{ m}$$

$$d = \frac{|\mathbf{M_O}|}{|\mathbf{F}|} \qquad d = 1.149 \text{ m}$$

F:[6,8,10] N $\mathbf{M_O}$**:[**−14,8,2] Nm, x=1 m

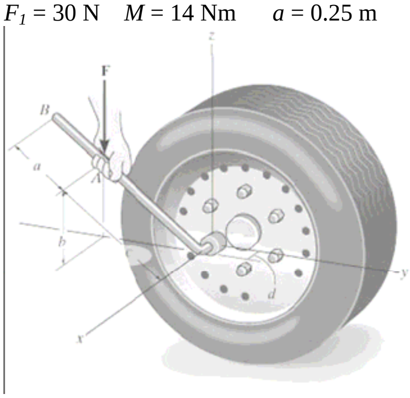
Problem 17
The force **F** is applied to the handle of the box wrench. Determine the component of the moment of this force about the z axis which is effective in loosening the bolt.



$$\mathbf{r} = \begin{pmatrix} c \\ -b \\ a \end{pmatrix} \qquad M_z = (\mathbf{r} \times \mathbf{F}) \cdot \mathbf{k} \qquad M_z = 62 \text{ lb} \cdot \text{in}$$

a = 3 in b = 8 in c = 2 in **F:[**8,−1,1] lb

Problem 18
The lug nut on the wheel of the automobile is to be removed using the wrench and applying the vertical force **F**. Assume that the cheater pipe AB is slipped over the handle of the wrench and the **F** force can be applied at any point and in any direction on the assembly. Determine if this force is adequate, provided a torque M about the *x-axis* is initially required to turn the nut.
Given:
F_1 = 30 N M = 14 Nm a = 0.25 m b = 0.3 m c = 0.5 m d = 0.1 m



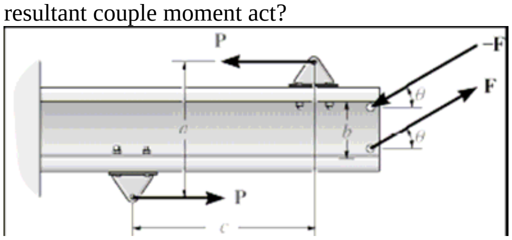
For M_{xmax}, apply force perpendicular to the handle and the x-axis.

$$M_x = F_1 \frac{a+c}{c} \sqrt{c^2 - b^2} \qquad M_{xmax} = F c$$

$$M_x = 18 \text{ N} \cdot \text{m} \qquad M_{xmax} = 15 \text{ N} \cdot \text{m}$$

$M_x > M$ Yes $M_{xmax} > M$ Yes

Problem 19
Two couples act on the beam. Determine the magnitude of **F** so that the resultant couple moment is M counterclockwise. Where on the beam does the resultant couple moment act?

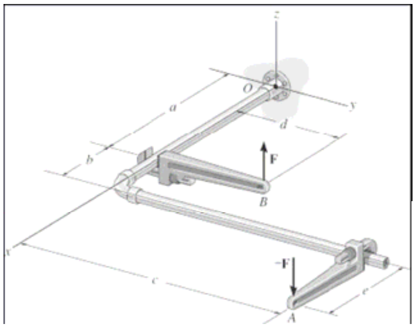


$$M_R = \Sigma M \qquad M = F b \cos(\theta) + P a \qquad F = \frac{M - P a}{b \cos(\theta)} \qquad F = 139 \text{ lb}$$

The resultant couple moment is a free vector. It can act at any point on the beam.

M = 450 lb ft P = 200 lb a = 1.5 ft b = 1.25 ft c = 2 ft θ = 30 deg

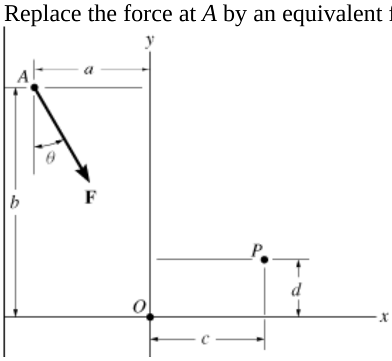
Problem 20
If the couple moment acting on the pipe has magnitude M, determine the magnitude F of the vertical force applied to each wrench.



$$\mathbf{r_{AB}} = \begin{pmatrix} -e-b \\ -c+d \\ 0 \end{pmatrix} \qquad \left|\mathbf{r_{AB}} \times (F\mathbf{k})\right| = M \qquad \text{Find}(F) \qquad F = 992.278\ \text{N}$$

Given:
M = 400 Nm a = 300 mm b = 150 mm c = 400 mm d = 200 mm e = 200 mm

Problem 21
Replace the force at A by an equivalent force and couple moment at point P.

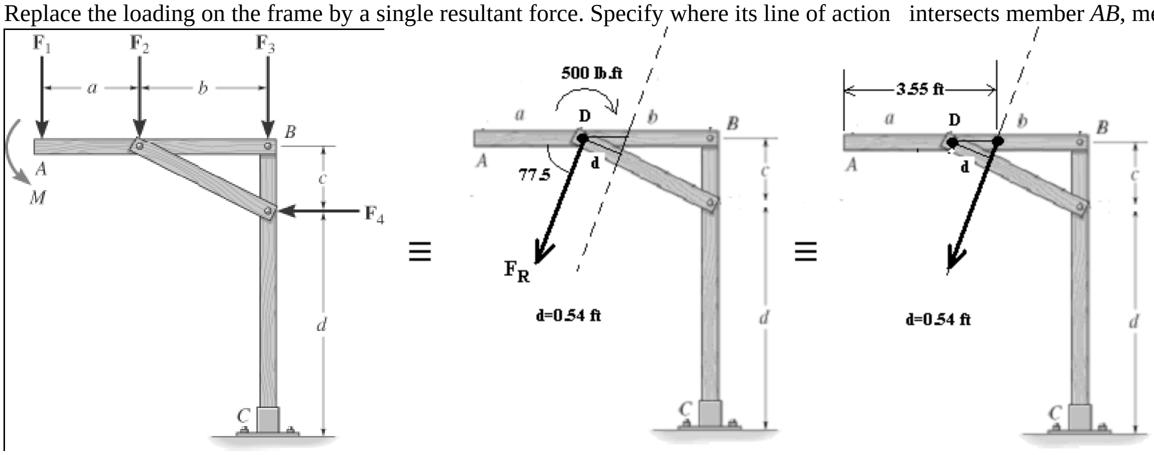


$$\mathbf{F} = F\begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{F} = \begin{pmatrix} 187.5 \\ -324.76 \\ 0 \end{pmatrix}\ \text{N}$$

$$\mathbf{M_P} = \begin{pmatrix} -a-c \\ b-d \\ 0 \end{pmatrix} \times \mathbf{F} \qquad \mathbf{M_P} = \begin{pmatrix} 0 \\ 0 \\ 736.538 \end{pmatrix}\ \text{N·m}$$

Given: F = 375 N a = 2 m b = 4 m c = 2 m d = 1 m θ = 30 deg

Problem 22
Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member AB, measured from A.



Given: M = 600 lb ft F_1 = 300 lb, F_2 = 200 lb, F_3 = 400 lb, F_4 = 200 lb a = 3 ft, b = 4 ft, c = 2 ft, d = 7 ft
Solution:

$F_{Rx} = -F_4$ $F_{Rx} = -200\ \text{lb}$

$F_{Ry} = -F_1 - F_2 - F_3$ $F_{Ry} = -900\ \text{lb}$

$$F = \sqrt{F_{Rx}^2 + F_{Ry}^2} \qquad F_R = 922\ \text{lb} \qquad \theta = \text{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right) \qquad \theta = 77.5\ \text{deg}$$

ΣM_D= (300)(3)- (400)(4)- (200)(2) + 600= -500 Lb.ft $\Sigma M_D = (F_R)(d)$ d= 0.54 ft